

The Four Color Theorem And Computer Based Proofs

The Four Color Theorem

- Every map can be colored with only four colors
 - This means that neighboring countries/areas are a different color
- History
 - Augustus de Morgan
- Works with every “Earthly” map including the world and the US
- Networks

Video explaining 7 colors or less

<https://youtu.be/NgbK43jB4rQ?t=501>



A Priori Knowledge

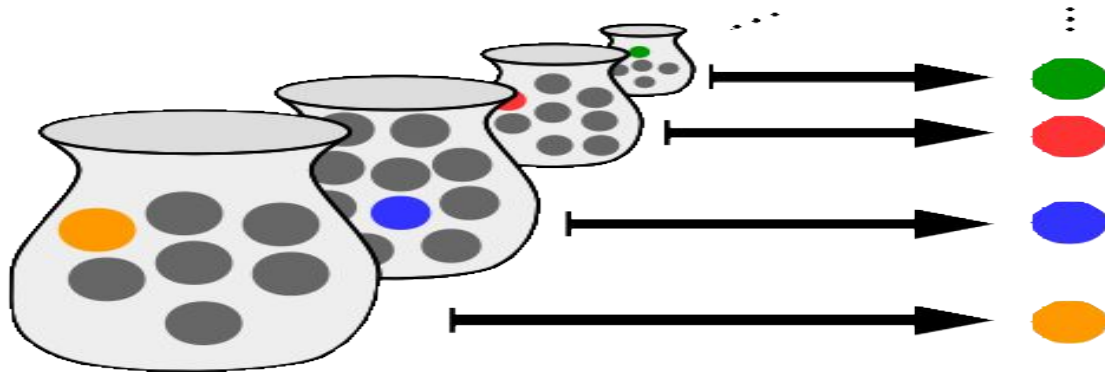
- A Priori knowledge is knowledge that is justified by evidence that lies outside the senses.
- In other words A Priori knowledge is knowledge that is justified by A Priori means.
- Many times this A Priori justification relies on our intuition of the world
- In Mathematics this justification comes from a certain set of axioms (Ex the Axiom of choice) and from the rules of logic.
- It is possible for current A Priori knowledge to be either proved or disproved by physical evidence.
- Ex Kant's belief in events with no cause.

Strong and Weak Forms of A Priori Knowledge

- Originally Kant thought that A Priori knowledge could not rely on any experience.
- However this restriction seems too strong and it is generally thought that A Priori knowledge can rely on experience to explain certain general concepts, as one cannot justify a statement which they do not understand.
- Ex Every home is a house.
- To understand this statement one has to know the concept of “house” and “home,” which might be learned through experience.
- However, once you know what the words in the statement mean then the statement does not require any physical evidence to be justified.

A Few More Examples

- If you know something, you believe it and it's true. Vs I know the Earth is the third planet from the sun.
- All bachelors are unmarried men. Vs all bachelors are taxed different from married men.
- The axiom of choice
- This example displays how A Priori knowledge can rely on a certain intuition, and might not be true or false.
- All pure Math knowledge (may encompass too much).



Is the knowledge gained from computer proofs A Priori?

Counter arguments:

- A computer based proof relies on the computer to be functioning properly, which is something that is only verified by physical action.
- Similarly relying on the functioning of any machine is essentially relying on the physical laws that govern each component of that machine.

Explanation

- However, normal mathematical proofs that are verified by a journal committee rely on the well-being of the evaluator's mind and brain.
- Thus this raises the question is there a difference in trusting a human to verify the logical steps of a proof vs trusting a computer to do so?
- Note that here we have the liberty to say trusting another human, as A Priori can be passed down to you without you actually knowing the full justification of the knowledge.
- Example of this is that most of us know the Pythagorean Theorem without actually knowing the proof of it.

Reasons To Doubt a Computer

- Errors in the programming of the computer. These can be seen as normally logically errors that happen in math.
- Errors in writing the code into the computer. This is similar to errors that occur in writing down a mathematical proof (Burg's paper assumes that pure math knowledge is A Priori).
- Defaults in the mechanical working of the computer.

The Inductive Argument

- An inductive argument is key concept on which the knowledge from a computer based proof relies on.
- This argument basically says that if a computer algorithm can be tested to perform correctly on a set smaller problems, then one can infer that the computer performs correctly on a larger problem which we cannot check.
- Can we accept this inductive argument and is it A priori justified?

So What?

- If we can trust the computers output on its work alone, then we can accept that its uncheckable work is A priori justified.
- In doing this, we accept a computer as a rational thinker or as something capable of contributing to the highest form of knowledge,
- Which is what early philosophers like Plato and Aristotle would say separates humans from other animals.

References

<https://drive.google.com/file/d/0B4me4PbBMBmONkMzTkIXRW83ZFE/view>

<https://plato.stanford.edu/entries/apriori/#WhaMealndPhrIndExp>